Chemical force
(TIGP nano-science 12/09/2009)

- Free energy and chemical potential
- Free energy and direction of reaction
- Dissociation
- Self-assembly of amphiphiles and interface

What is force and generalized forces?

- The variation of energy with respect to position

  $F = -dU/dx$ (a single particle situation)

- The variation of system energy with respect to change of volume

  $$P = -dE/dV$$

- The variation of system energy with respect to change of configuration

  $$T = -dE/dS$$

- The variation of system energy with respect to exchange of particle

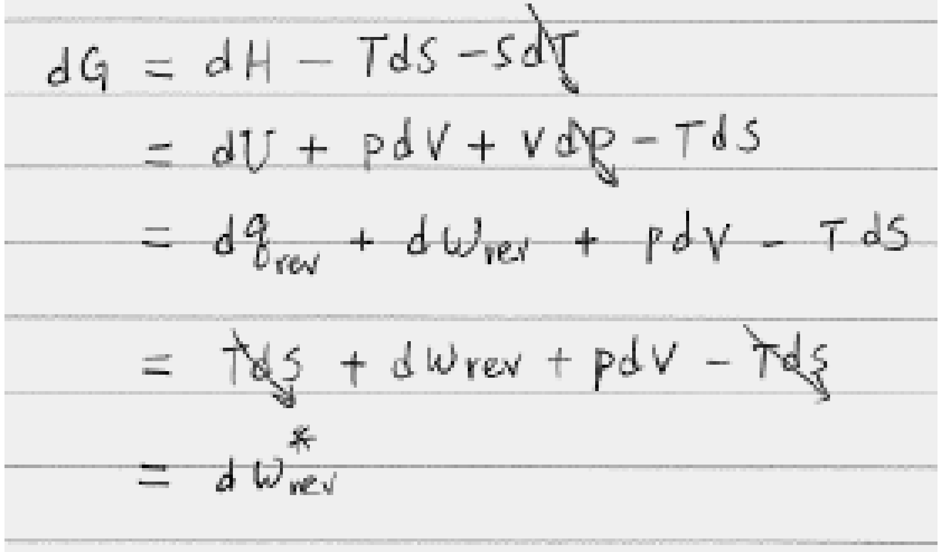
  $$\mu = -dE/dN$$

Free energy

Entropy	S
Enthalpy	$H = U+PV$
Helmholtz	$F = U-TS$
Gibbs	$G = U+PV-TS=H -TS$
Grand Ensemble	$G = H -TS+\mu N$

Free Energy change and effective work

Theorem: At constant temperature and constant pressure, through reversible process, the change of free energy equals effective (non-expansive) work.



$$\begin{aligned} dG &= dH - TdS - Sd\cancel{T} \\ &= dU + pdV + Vd\cancel{P} - TdS \\ &= dq_{rev} + dw_{rev} + pdV - TdS \\ &= \cancel{TdS} + dw_{rev} + pdV - \cancel{TdS} \\ &= dw^{*}_{rev} \end{aligned}$$

$$\int dG \rightarrow \Delta G = w^{*}_{rev}$$

Free energy and chemical potential

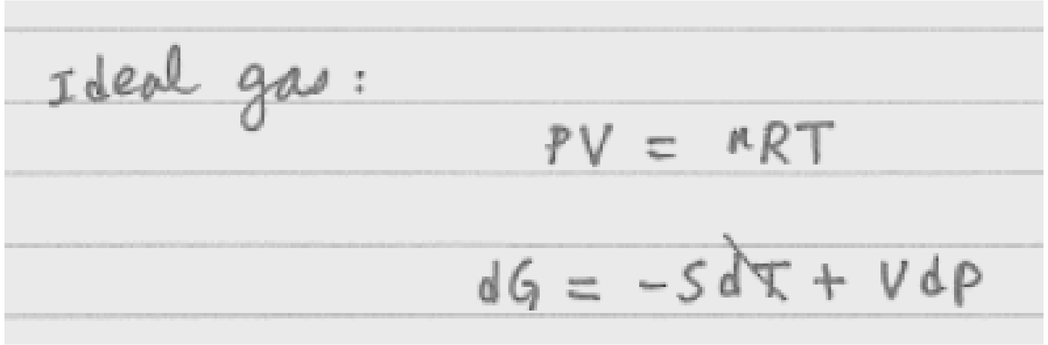
$$G = G(T, P) \longrightarrow G(T, P, n_A, n_B, \cdots)$$

e.g. Phase equilibrium between water and vapor

Define $$\mu_A \equiv \left(\frac{\partial G}{\partial n_A}\right)_{T,P,n_j \neq n_A}$$

Chemical potential: the change of free energy upon addition into or subtraction of a particle from the system.

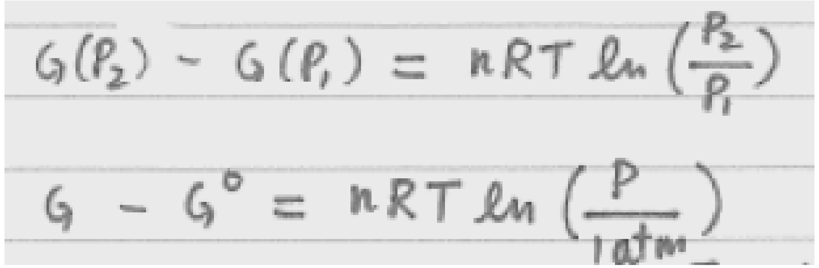
Dependence of chemical potential on pressure



Ideal gas:

$$PV = nRT$$

$$dG = -S\cancel{dT} + Vdp$$



$$G(P_2) - G(P_1) = nRT\ln\left(\frac{P_2}{P_1}\right)$$

$$G - G^0 = nRT\ln\left(\frac{P}{1atm}\right)$$

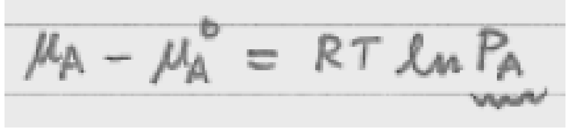
G^0: reference point; standard free energy

Divide by n

$$\mu - \mu^0 = RT\ln P$$

μ^0: standard chemical potential

Species A



$$\mu_A - \mu_A^0 = RT\ln \underline{P_A}$$

P_A: partial pressure of A

By analogy $\mu_A - \mu_A^0 = RT * \ln(n_A)$

Dissociation

- Chemical equilibrium of reversible reactions

- Assembly and disassembly of molecules

- Equilibrium constant and Free energy

Chemical potential and directionality of the chemical reaction

$$G = n_A \mu_A + n_B \mu_B + n_C \mu_C + n_D \mu_D + \cdots$$

$$aA + bB \rightleftharpoons cC + dD$$

$$dG = \mu_A \, dn_A + \mu_B \, dn_B + \mu_C \, dn_C + \mu_D \, dn_D$$

constant T and P

$$\frac{dn_A}{a} = \frac{dn_B}{b} = -\frac{dn_C}{c} = -\frac{dn_D}{d} = -d\underline{\alpha}$$

stoichiometry constraint

$$dG = -[\underline{(a\mu_A + b\mu_B) - (c\mu_C + d\mu_D)}]\, d\alpha$$

We know that if the reaction goes to the right, dα must be positive. We also know that if the reaction goes to right, dG must be negative.

Chemical equilibrium constant

$$aA + bB \rightleftharpoons cC + dD$$

Gibbs free energy of reactants: $a\mu_A + b\mu_B$

Gibbs free energy of products: $c\mu_C + d\mu_D$

ΔG of reaction is

$$c\mu_C + d\mu_D - a\mu_A - b\mu_B$$

$$= (c\mu_C^\circ + d\mu_D^\circ - a\mu_A^\circ - b\mu_B^\circ) + RT\cdot(c\ln n_C + d\ln n_D - a\ln n_A - b\ln n_B$$

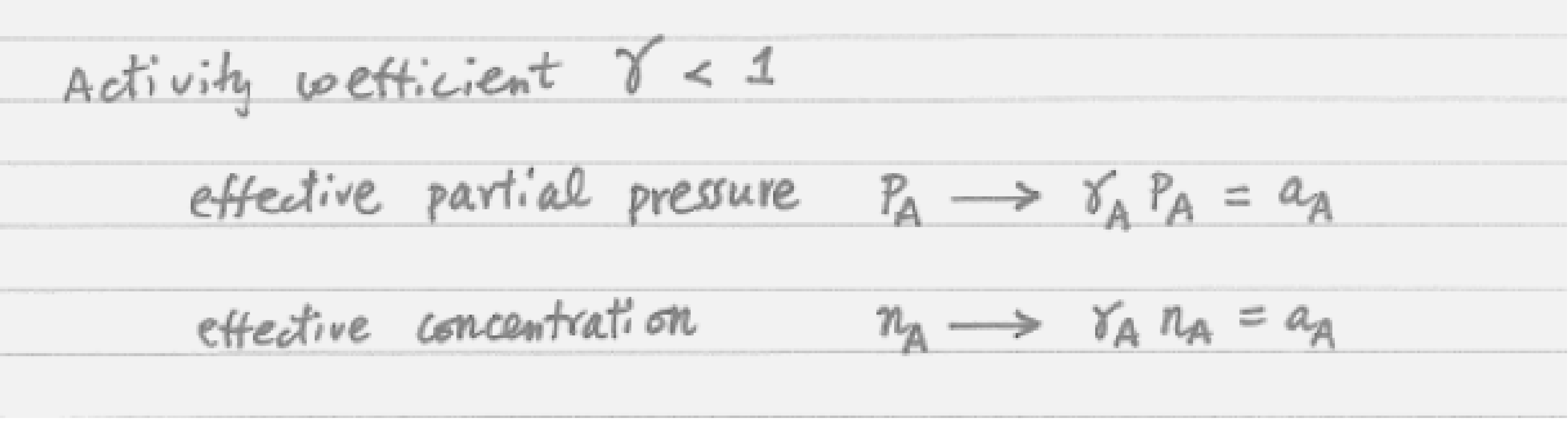
$$= \Delta G^\circ + RT\left[\ln\left(\frac{n_C^c \cdot n_D^d}{n_A^a \cdot n_B^b}\right)\right]$$

At equilibrium, $\Delta G = 0$

$$\rightarrow \Delta G^\circ = -RT\left[\ln\left(\frac{(n_C^{eq})^c \cdot (n_D^{eq})^d}{(n_A^{eq})^a \cdot (n_B^{eq})^b}\right)\right] = -RT\ln K$$

most useful thermodynamics equation

Non-ideal system

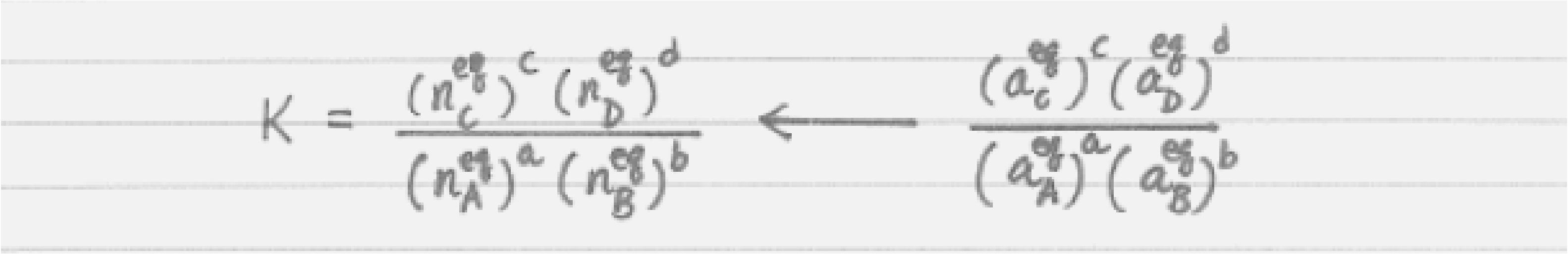


Activity coefficient $\gamma < 1$

effective partial pressure $P_A \longrightarrow \gamma_A P_A = a_A$

effective concentration $n_A \longrightarrow \gamma_A n_A = a_A$

When concentration is low, γ is close to 1.



$$K = \frac{(n_C^{eq})^c (n_D^{eq})^d}{(n_A^{eq})^a (n_B^{eq})^b} \longleftarrow \frac{(a_C^{eq})^c (a_D^{eq})^d}{(a_A^{eq})^a (a_B^{eq})^b}$$

Dependence of chemical equilibrium on temperature

$$K(T_1) \longrightarrow K(T_2)$$

van't Hoff equation

$$\frac{d(G/T)}{dT} = -\frac{H}{T^2}$$

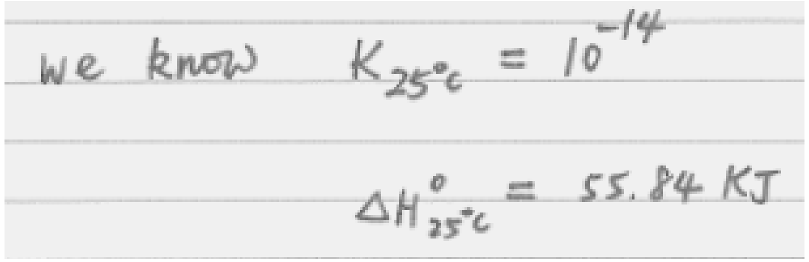
$$\rightarrow \frac{d(\Delta G^\circ/T)}{dT} = -\frac{\Delta H^\circ}{T^2}$$

$$R\,\frac{d(\ln K)}{dT} = \frac{\Delta H^\circ}{T^2}$$

$$\rightarrow \frac{d\ln K}{d\left(\frac{1}{T}\right)} = -\frac{\Delta H^\circ}{R}$$

Application to water

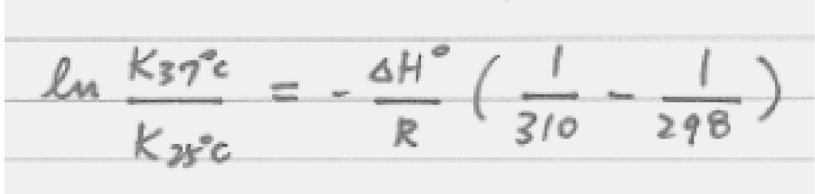
pH of water or buffer solution



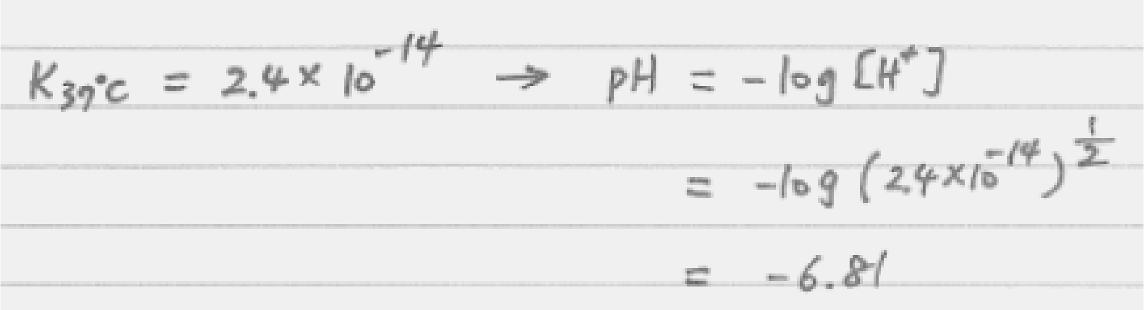
we know $K_{25°c} = 10^{-14}$

$\Delta H^{\circ}_{25°c} = 55.84\ KJ$

consult a biophysical text book



$$\ln \frac{K_{37°c}}{K_{25°c}} = -\frac{\Delta H^{\circ}}{R}\left(\frac{1}{310} - \frac{1}{298}\right)$$



$K_{37°c} = 2.4 \times 10^{-14} \rightarrow pH = -\log[H^{+}]$

$= -\log(24 \times 10^{-14})^{\frac{1}{2}}$

$= -6.81$

Application to Tris buffer

PK = 8.3 at 20°C

$\Delta PK / \Delta T = -0.029\ K^{o-1}$

26.8 ml 0.2 M HCl mixed with 50 ml 0.2 M Tris Add water 200 ml

pH at 20°C ?

pH at 37°C ?

buffer capacity change of pH by adding 1 mmol strong acid or base to 1 L 50 mM Tris (20 C)

Solution for 20 C

	$\mathrm{Tris\cdot H^+}$ $\rightleftharpoons$	Tris +	$\mathrm{H^+}$	
initial	0	10 (mmol)	5.36 (mmol)	/ 0.2 l
final	5.36	10−5.36	**small quantities**	
initial	**00.0 mM**	**50.0 mM**	**small quantities**	
final	**26.8 mM**	**23.2 mM**	**small quantities**	

$$K_{Tris} = \frac{[H^+]\cdot[Tris]}{[Tris\cdot H^+]}$$

$$\underset{\overset{\parallel}{pK_{Tris}}}{-\log K_{Tris}} = \underset{\overset{\parallel}{pH}}{-\log[H^+]} - \log\frac{[Tris]}{[Tris\cdot H^+]}$$

$$8.3 = \mathbf{8.2} + 0.063$$

Solution for 37 C

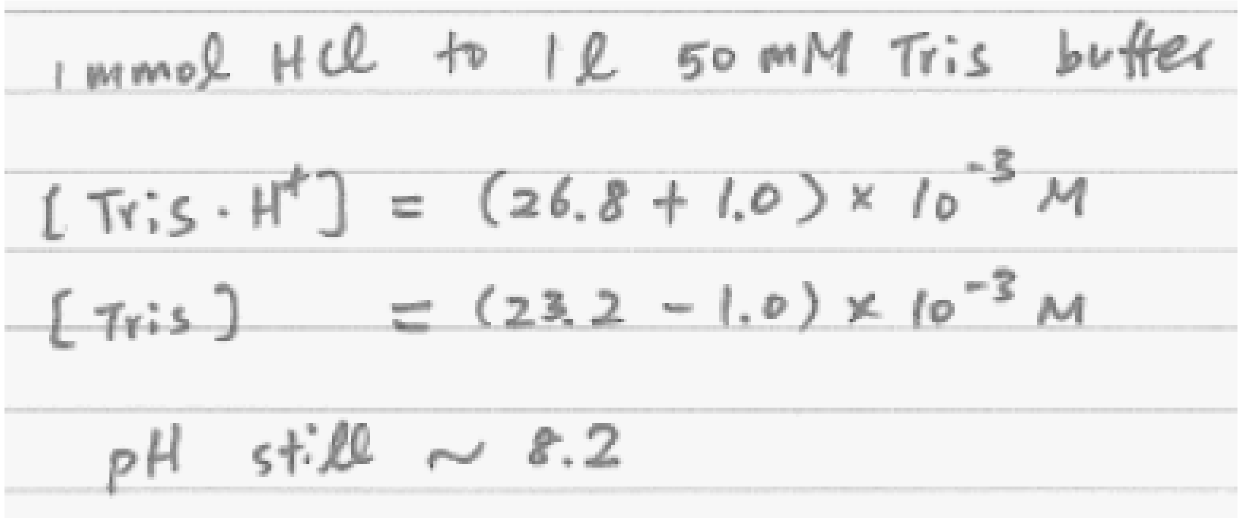
$$K_{Tris} = \frac{[H^+]\cdot[Tris]}{[Tris\cdot H^+]}$$

$$\underset{\underset{pK_{Tris}}{\parallel}}{-\log K_{Tris}} = \underset{\underset{pH}{\parallel}}{-\log[H^+]} - \log\frac{[Tris]}{[Tris\cdot H^+]}$$

$$pK_{Tris@37} = \underset{8.3}{pK_{Tris@20}} - (0.029)(37-20) = 7.8$$

$$\underset{\underset{\underset{7.8}{pK_{Tris}}}{\parallel}}{-\log K_{Tris}} = \underset{\underset{\underset{7.7}{pH}}{\parallel}}{-\log[H^+]} - \log\frac{[Tris]}{[Tris\cdot H^+]}$$

Solution for buffering power



1 mmol HCl to 1 l 50 mM Tris buffer

$[\text{Tris}\cdot\text{H}^+] = (26.8 + 1.0) \times 10^{-3}$ M

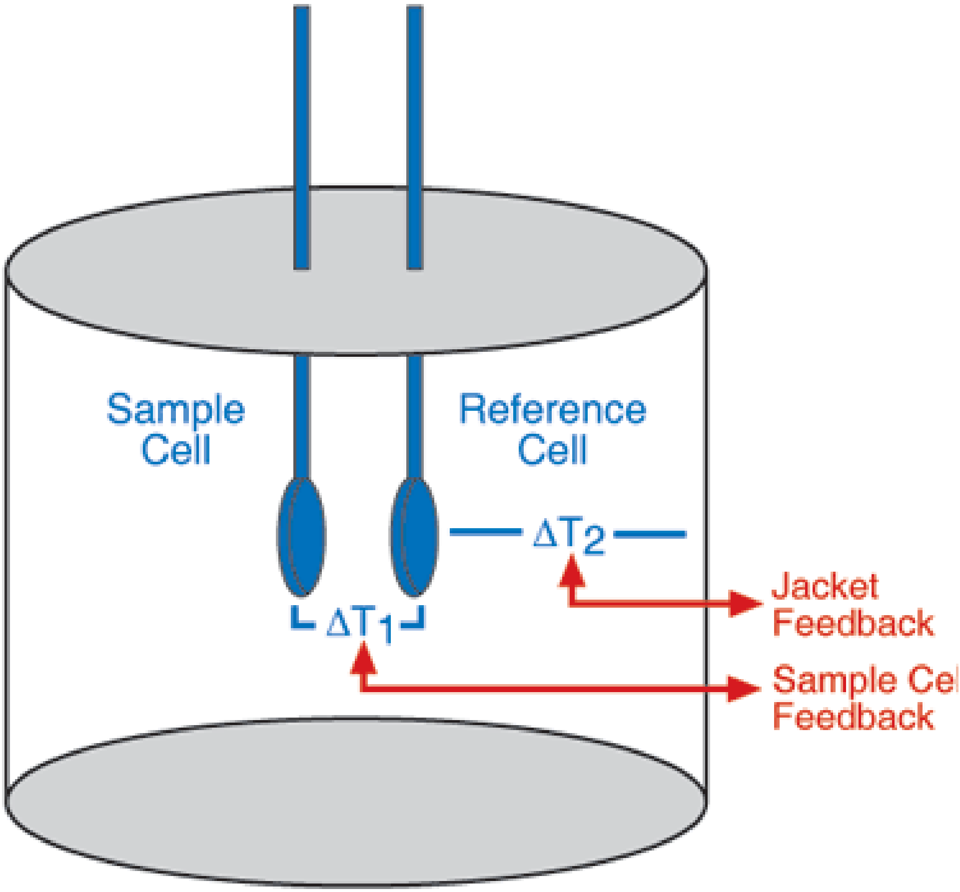
$[\text{Tris}] = (23.2 - 1.0) \times 10^{-3}$ M

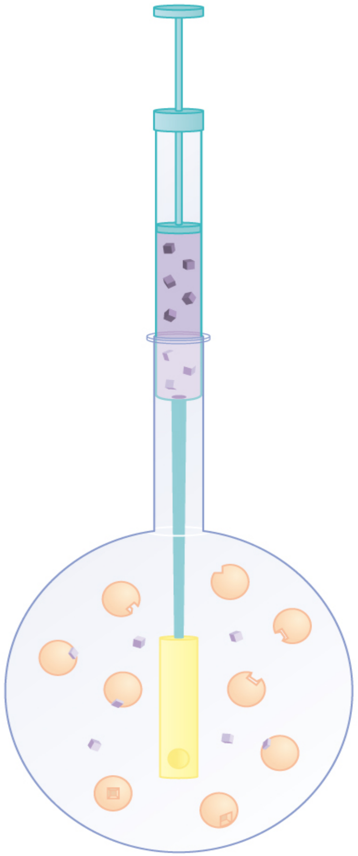
pH still ~ 8.2

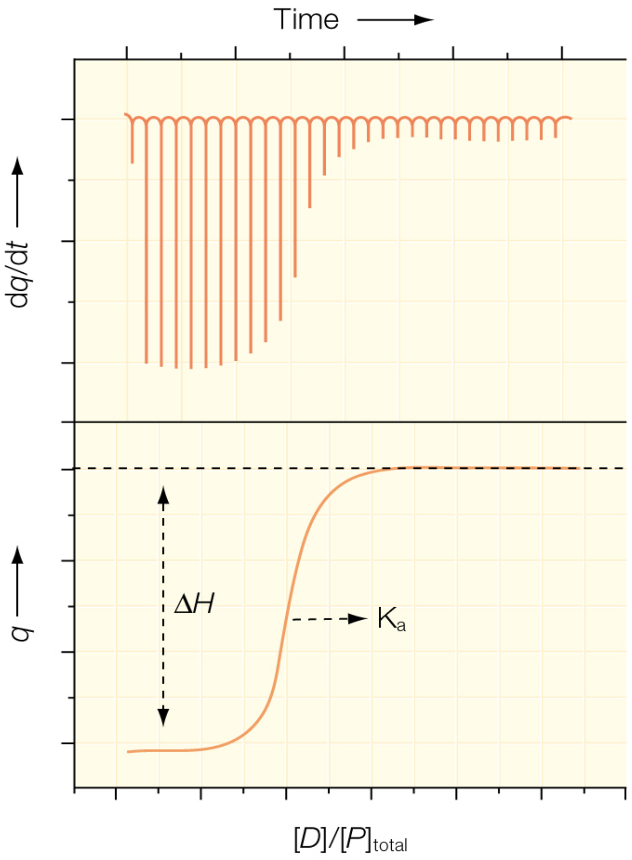
What if we do the same thing to 5 mM Tris instead of 50 mM Tris?

Isothermal Titration Calorimetry

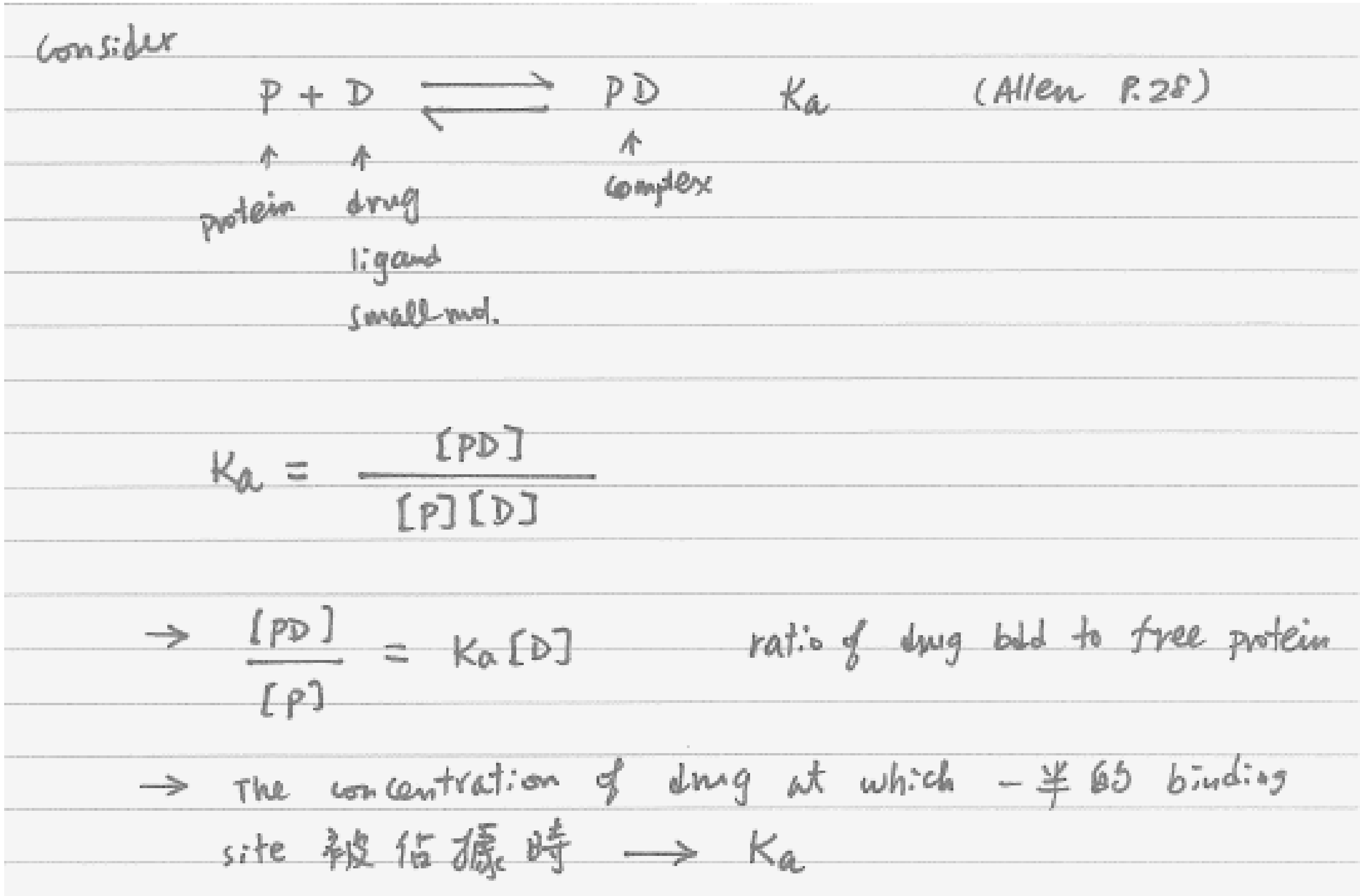
http://www.microcal.com/technology/itc-animation.asp







Information from ITC data



consider

$$P + D \rightleftharpoons PD \quad K_a \quad \text{(Allen P.28)}$$

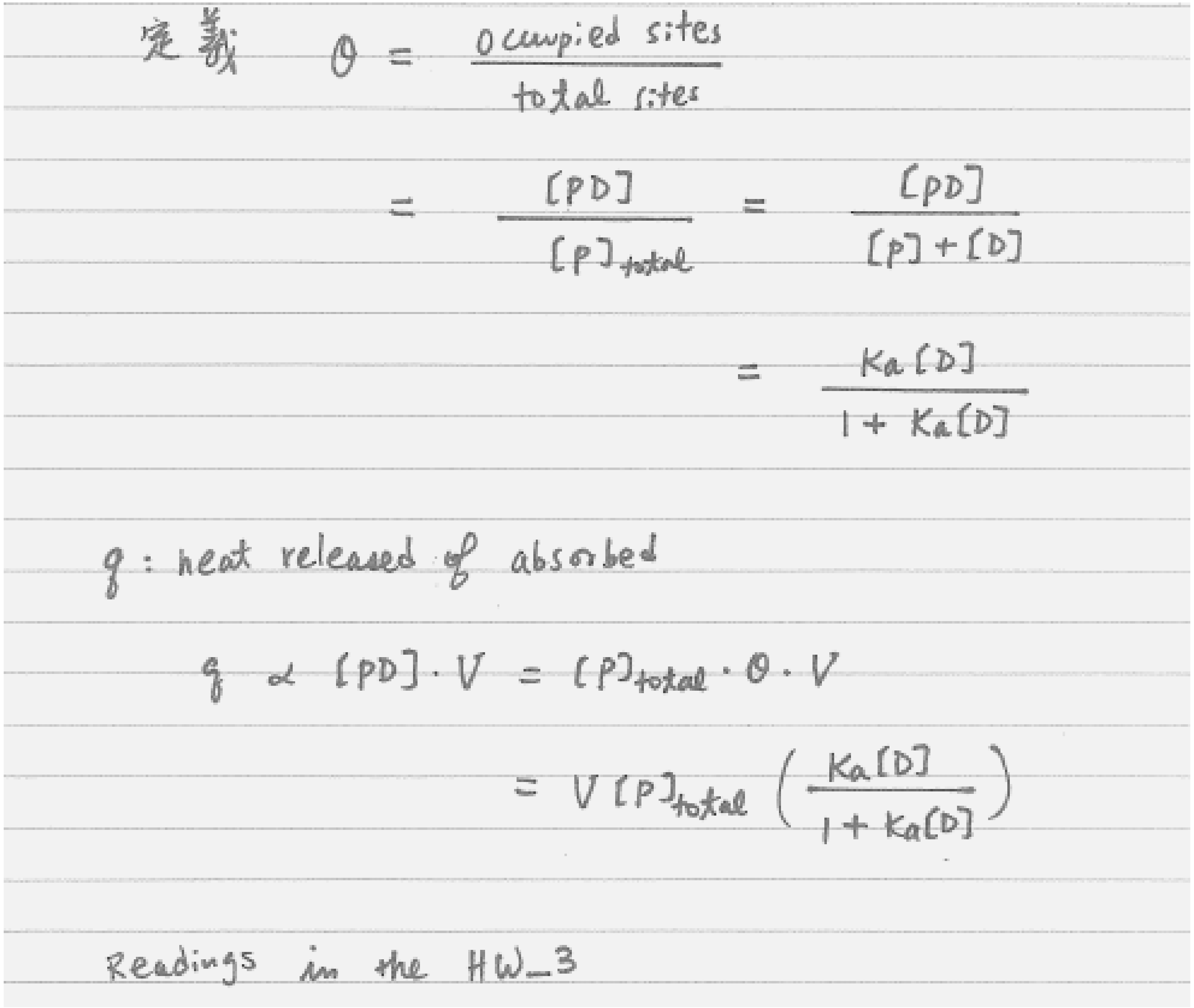
$\uparrow$ protein $\uparrow$ drug / ligand / small mol. $\uparrow$ complex

$$K_a = \frac{[PD]}{[P][D]}$$

$$\rightarrow \frac{[PD]}{[P]} = K_a[D] \quad \text{ratio of drug bdd to free protein}$$

→ The concentration of drug at which 一半的 binding site 被佔據時 → K_a

The thermal curve and the binding



定義 $\theta = \dfrac{\text{occupied sites}}{\text{total sites}}$

$$= \frac{[PD]}{[P]_{total}} = \frac{[PD]}{[P]+[D]}$$

$$= \frac{K_a[D]}{1+K_a[D]}$$

q : heat released of absorbed

$$q \propto [PD]\cdot V = [P]_{total}\cdot\theta\cdot V$$

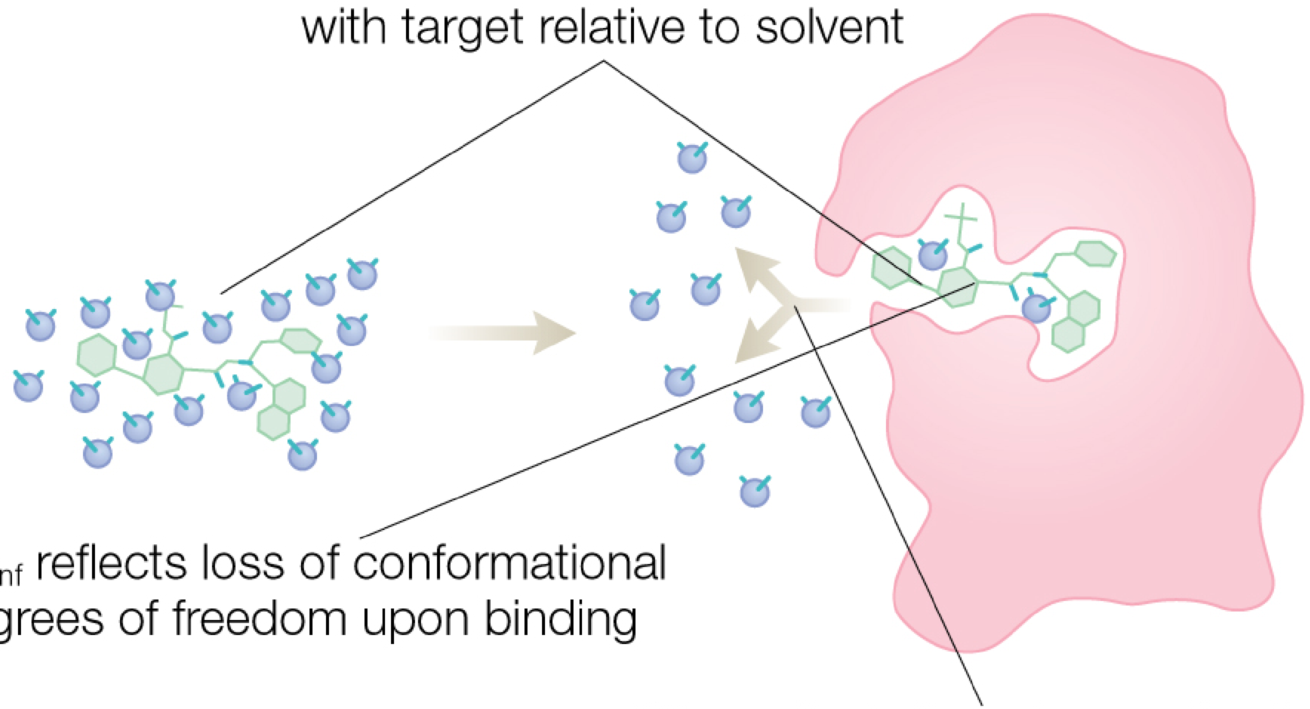
$$= V[P]_{total}\left(\frac{K_a[D]}{1+K_a[D]}\right)$$

Readings in the HW_3

The most important PChem formula for biochemists

$\Delta G = -RT \ln \underset{\sim\sim}{K_a}$ (strange or not ???)

$\Delta G = \Delta H - T\Delta S$ (Figure 3.6 Allen P.57)

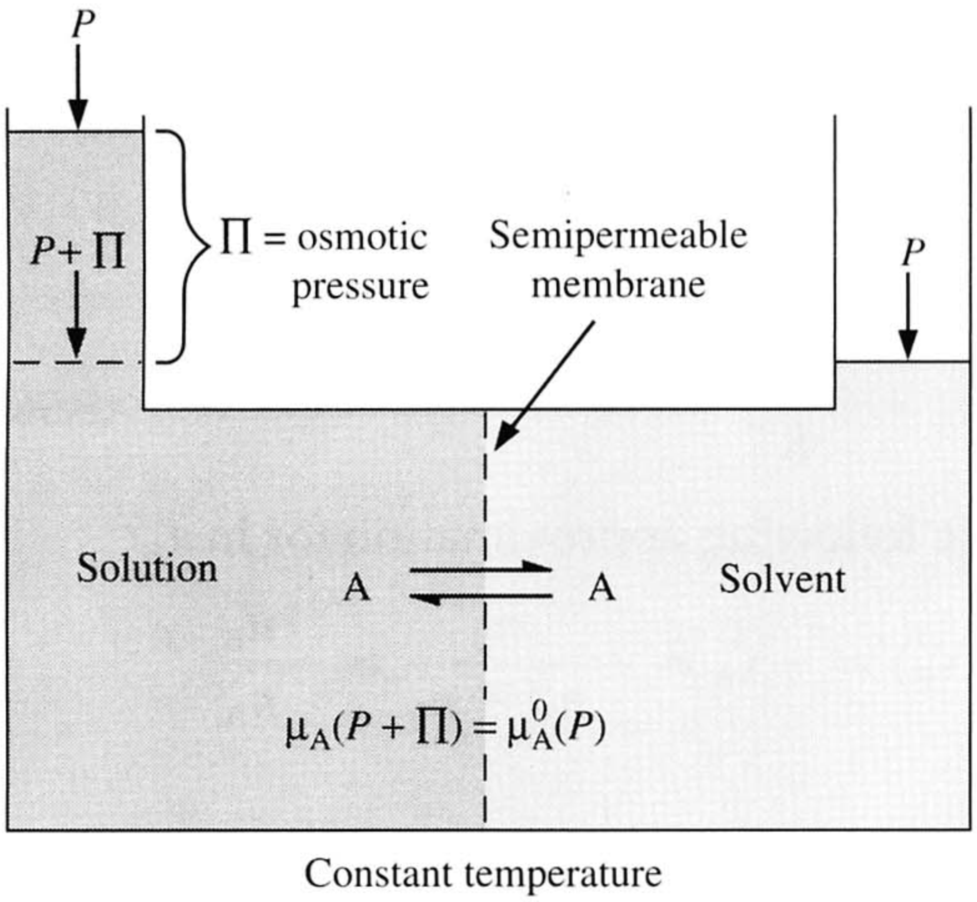


ΔH reflects strength of interaction with target relative to solvent

ΔS_{conf} reflects loss of conformational degrees of freedom upon binding

ΔS_{solv} reflects the release of water upon binding

Osmotic pressure can be quantified



◀ FIGURE 5.25

Osmometer showing conditions at equilibrium. A is a solvent molecule.

Deriving osmotic pressure

$$\mu_A(\text{solution}, P+\Omega) = \mu_A(\text{solvent}, P)$$

$$\rightarrow \quad \mu_A(\text{solution}, 1+\Omega) = \mu_A^\circ \qquad (\text{set } P = 1 \text{ atm})$$

$$\|$$

$$\mu_A^\circ + RT \ln a_A$$

Recall that the pressure dependence of Gibbs free energy gives that

$$G(P_2) - G(P_1) \cong V(P_2 - P_1)$$

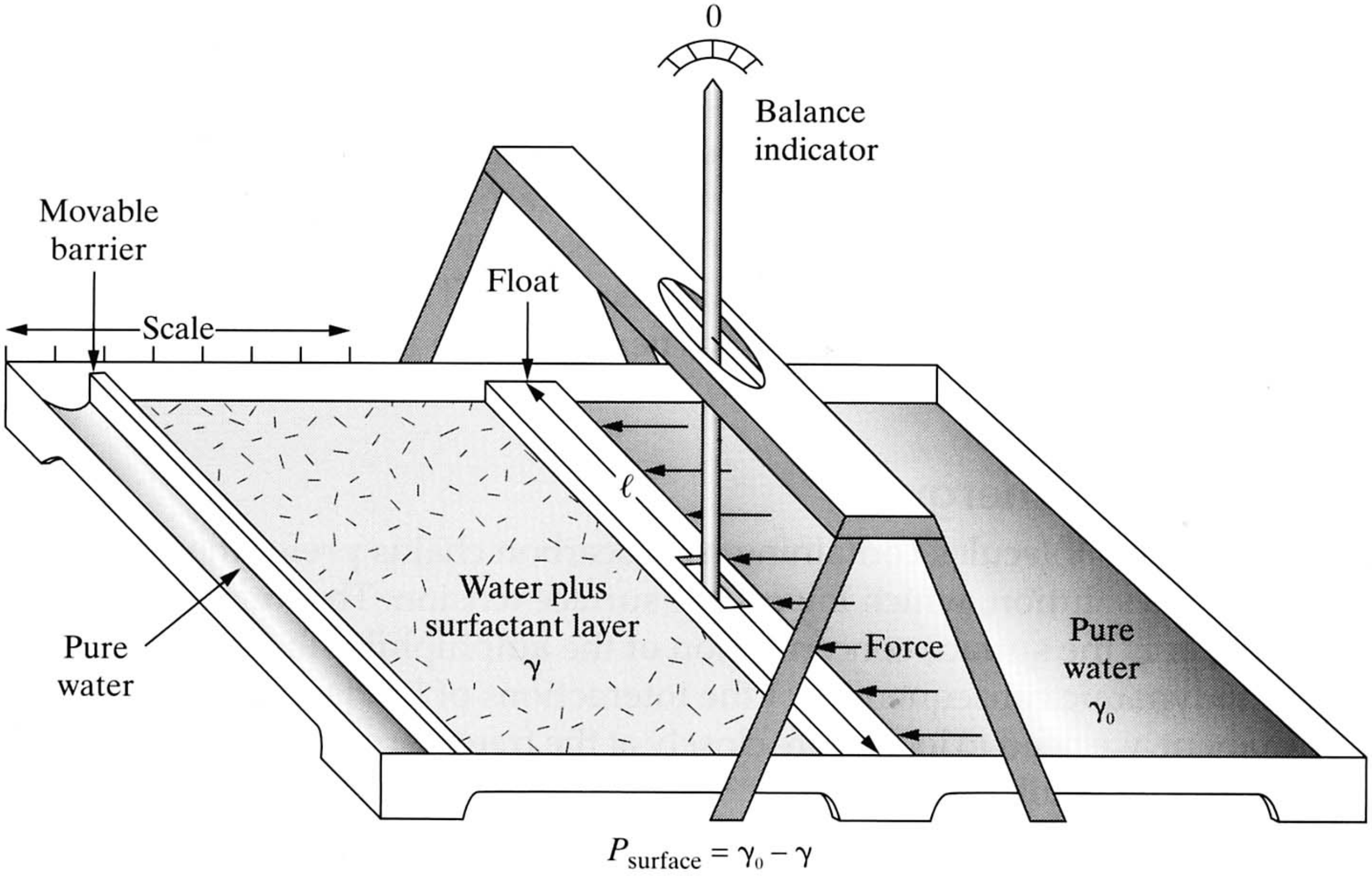
$$\mu_A - \mu_A^\circ = -\bar{V}_A \cdot \Omega \qquad (\bar{V}_A \text{ specific volume of solvent})$$

$$\ln a_A \sim \ln X_A = \ln(1 - X_B) \sim -X_B \sim \frac{n_B}{n_A} \qquad (\text{dilute solution})$$

$$\Omega = \frac{RT}{n_A \bar{V}_A} \cdot n_B = c \cdot RT$$

C: concentration of solute

LB film measurement



▲ FIGURE 5.18

A Langmuir film balance measures surface tension. The force per unit length F/ℓ required to compress a surface containing a surfactant layer is known as the surface pressure P_{surface}. The float is connected to a torsion wire (not shown) to measure the force.

Critical Micelle Concentration (CMC)